

```

⊢ ∀[A,B:ℙ]. {A ∨ B ⇔ ¬((¬A) ∧ (¬B))}
|
BY (D 0 THENA Auto)
| \
| 1. A: ℙ
| ⊢ ∀[B:ℙ]. {A ∨ B ⇔ ¬((¬A) ∧ (¬B))}
| |
1 BY (D 0 THENA Auto)
| | \
| | 2. B: ℙ
| | ⊢ {A ∨ B ⇔ ¬((¬A) ∧ (¬B))}
| | |
1 2 BY RepeatFor 4 ((D 0 THENA Auto))
| | | \
| | | 3. A ∨ B
| | | ⊢ {¬((¬A) ∧ (¬B))}
| | | |
1 2 3 BY ElimClassical
| | | | \
| | | | ⊢ ¬((¬A) ∧ (¬B))
| | | | |
1 2 3 4 BY (D 0 THENA Auto)
| | | | |
| | | | 4. (¬A) ∧ (¬B)
| | | | ⊢ False
| | | | |
1 2 3 4 BY D 4
| | | | |
| | | | 4. ¬A
| | | | 5. ¬B
| | | | ⊢ False
| | | | |
1 2 3 4 BY D 3
| | | | | \
| | | | | 3. A
| | | | | ⊢ False
| | | | | |
1 2 3 4 5 BY D 4
| | | | | |
| | | | | 4. ¬B
| | | | | ⊢ A
| | | | | |
1 2 3 4 5 BY Hypothesis
| | | | | \
| | | | | 3. B
| | | | | ⊢ False
| | | | | |
1 2 3 4 BY D 5
| | | | | |
| | | | | ⊢ B
| | | | | |
1 2 3 4 BY Hypothesis
| | | | | \
| | | | 4. x: Unit
| | | | ⊢ (¬A) ∧ (¬B) ∈ ℙ
| | | | |

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1 2 3   BY Auto
| | \
| | 3.  $\neg((\neg A) \wedge (\neg B))$ 
| |  $\vdash \{A \vee B\}$ 
| | |
1 2   BY (ClassicalContradiction THENA Auto)
| | |
| | 4.  $\neg(A \vee B)$ 
| |  $\vdash \{A \vee B\}$ 
| | |
1 2   BY D 3
| | |
| | 3.  $\neg(A \vee B)$ 
| |  $\vdash (\neg A) \wedge (\neg B)$ 
| | |
1 2   BY D 0
| | | \
| | |  $\vdash \neg A$ 
| | | |
1 2   3 BY (D 0 THENA Auto)
| | | |
| | | | 4. A
| | | |  $\vdash \text{False}$ 
| | | |
1 2   3 BY D 3
| | | |
| | | | 3. A
| | | |  $\vdash A \vee B$ 
| | | |
1 2   3 BY (OrLeft THENA Auto)
| | | |
| | | |  $\vdash A$ 
| | | |
1 2   3 BY Hypothesis
| | | \
| | |  $\vdash \neg B$ 
| | | |
1 2   BY (D 0 THENA Auto)
| | | |
| | | | 4. B
| | | |  $\vdash \text{False}$ 
| | | |
1 2   BY D 3
| | | |
| | | | 3. B
| | | |  $\vdash A \vee B$ 
| | | |
1 2   BY (OrRight THENA Auto)
| | | |
| | | |  $\vdash B$ 
| | | |
1 2   BY Hypothesis
| | | \
| | | 2. B:  $\mathbb{P}$ 
| | | 3.  $\{x:\text{Unit} \mid A \vee B \iff \neg((\neg A) \wedge (\neg B))\}$ 
| | |  $\vdash \text{Ax} \in \{x:\text{Unit} \mid A \vee B \iff \neg((\neg A) \wedge (\neg B))\}$ 
| | |

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```
1  BY Auto
  \
  1. A:  $\mathbb{P}$ 
  2. B:  $\mathbb{P}$ 
  3.  $\{x:\text{Unit} \mid A \vee B \iff \neg((\neg A) \wedge (\neg B))\}$ 
 $\vdash Ax \in \{x:\text{Unit} \mid A \vee B \iff \neg((\neg A) \wedge (\neg B))\}$ 
  |
  BY Auto
```