

```

┆ ∀[T:Type]. ∀[A:T → ℙ]. {¬(∀x:T. {A x})} ⇔ ∃x:T. (¬(A x))}
|
BY (D 0 THENA Auto)
| \
| 1. T: Type
| ┆ ∀[A:T → ℙ]. {¬(∀x:T. {A x})} ⇔ ∃x:T. (¬(A x))}
| |
1 BY (D 0 THENA Auto)
| | \
| | 2. A: T → ℙ
| | ┆ {¬(∀x:T. {A x})} ⇔ ∃x:T. (¬(A x))}
| | |
1 2 BY RepeatFor 4 ((D 0 THENA Auto))
| | | \
| | | 3. ¬(∀x:T. {A x})
| | | ┆ {∃x:T. (¬(A x))}
| | | |
1 2 3 BY (ClassicalContradiction THENA Auto)
| | | |
| | | 4. ¬(∃x:T. (¬(A x)))
| | | ┆ {∃x:T. (¬(A x))}
| | | |
1 2 3 BY D 3
| | | |
| | | 3. ¬(∃x:T. (¬(A x)))
| | | ┆ ∀x:T. {A x}
| | | |
1 2 3 BY (D 0 THENA Auto)
| | | |
| | | 4. x: T
| | | ┆ {A x}
| | | |
1 2 3 BY (ClassicalContradiction· THENA Auto)
| | | |
| | | 5. ¬(A x)
| | | ┆ {A x}
| | | |
1 2 3 BY D 3
| | | |
| | | 3. x: T
| | | 4. ¬(A x)
| | | ┆ ∃x:T. (¬(A x))
| | | |
1 2 3 BY (InstConcl [x]. THENA Auto)
| | | |
| | | ┆ ¬(A x)
| | | |
1 2 3 BY Hypothesis
| | \
| | 3. ∃x:T. (¬(A x))
| | ┆ {¬(∀x:T. {A x})}
| | |
1 2 BY (ElimClassical THENA Auto)
| | |
| | ┆ ¬(∀x:T. {A x})
| | |

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1 2 BY (D 0 THENA Auto)
| | |
| | 4.  $\forall x:T. \{A x\}$ 
| |  $\vdash$  False
| | |
1 2 BY D 3
| | |
| | 3.  $x: T$ 
| | 4.  $\neg(A x)$ 
| | 5.  $\forall x:T. \{A x\}$ 
| |  $\vdash$  False
| | |
1 2 BY (InstHyp [ $x$ ] 5. THENA Auto)
| | |
| | 6.  $\{A x\}$ 
| |  $\vdash$  False
| | |
1 2 BY D 6
| | |
| | 6.  $x@0: \text{Unit}$ 
| | 7.  $A x$ 
| |  $\vdash$  False
| | |
1 2 BY D 4
| | |
| | 4.  $\forall x:T. \{A x\}$ 
| | 5.  $x@0: \text{Unit}$ 
| | 6.  $A x$ 
| |  $\vdash A x$ 
| | |
1 2 BY Hypothesis
| \
| 2.  $A: T \rightarrow \mathbb{P}$ 
| 3.  $\{x:\text{Unit} \mid \neg(\forall x:T. \{A x\}) \iff \exists x:T. (\neg(A x))\}$ 
|  $\vdash Ax \in \{x:\text{Unit} \mid \neg(\forall x:T. \{A x\}) \iff \exists x:T. (\neg(A x))\}$ 
| |
1 BY Auto
\
1.  $T: \text{Type}$ 
2.  $A: T \rightarrow \mathbb{P}$ 
3.  $\{x:\text{Unit} \mid \neg(\forall x:T. \{A x\}) \iff \exists x:T. (\neg(A x))\}$ 
 $\vdash Ax \in \{x:\text{Unit} \mid \neg(\forall x:T. \{A x\}) \iff \exists x:T. (\neg(A x))\}$ 
|
BY Auto

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