

```

┆ ∀[T:Type]. (T ⇒ (∀[A:T → ℙ]. ∀[B:ℙ]. {∃x:T. ((A x) ⇒ B) ⇔ (∀x:T. {A x} ⇒ B)}))
|
BY Auto
|
1. T: Type
2. T
3. A: T → ℙ
4. B: ℙ
┆ {∃x:T. ((A x) ⇒ B) ⇔ (∀x:T. {A x} ⇒ B)}
|
BY RepeatFor 4 ((D 0 THENA Auto))
| \
| 5. ∃x:T. ((A x) ⇒ B)
| ┆ {(∀x:T. {A x} ⇒ B)}
| |
1 BY RepeatFor 2 ((D 0 THENA Auto))
| |
| 6. ∀x:T. {A x}
| ┆ {B}
| |
1 BY D 5
| |
| 5. x: T
| 6. (A x) ⇒ B
| 7. ∀x:T. {A x}
| ┆ {B}
| |
1 BY (InstHyp [x] 7. THENA Auto)
| |
| 8. {A x}
| ┆ {B}
| |
1 BY ExposeClassical
| |
| 8. A x
| ┆ {B}
| |
1 BY D 6
| | \
| | 6. ∀x:T. {A x}
| | 7. A x
| | ┆ A x
| | |
1 2 BY Hypothesis
| \
| 6. ∀x:T. {A x}
| 7. A x
| 8. B
| ┆ {B}
| |
1 BY ElimClassical
| |
| ┆ B
| |
1 BY Hypothesis
\

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5.  $(\forall x:T. \{A\ x\}) \Rightarrow B$ 
 $\vdash \{\exists x:T. ((A\ x) \Rightarrow B)\}$ 
|
BY (ClassicalContradiction THENA Auto)
|
6.  $\neg(\exists x:T. ((A\ x) \Rightarrow B))$ 
 $\vdash \{\exists x:T. ((A\ x) \Rightarrow B)\}$ 
|
BY (ElimClassical THENA Auto)
|
 $\vdash \exists x:T. ((A\ x) \Rightarrow B)$ 
|
BY D 5
| \
| 5.  $\neg(\exists x:T. ((A\ x) \Rightarrow B))$ 
|  $\vdash \forall x:T. \{A\ x\}$ 
| |
1 BY (D 0 THENA Auto)
| |
| 6.  $x: T$ 
|  $\vdash \{A\ x\}$ 
| |
1 BY (ClassicalContradiction THENA Auto)
| |
| 7.  $\neg(A\ x)$ 
|  $\vdash \{A\ x\}$ 
| |
1 BY D 5
| |
| 5.  $x: T$ 
| 6.  $\neg(A\ x)$ 
|  $\vdash \exists x:T. ((A\ x) \Rightarrow B)$ 
| |
1 BY (InstConcl [ $\lceil x \rceil$ ]. THENA Auto)
| |
|  $\vdash (A\ x) \Rightarrow B$ 
| |
1 BY (D 0 THENA Auto)
| |
| 7.  $A\ x$ 
|  $\vdash B$ 
| |
1 BY D 6
| |
| 6.  $A\ x$ 
|  $\vdash A\ x$ 
| |
1 BY Hypothesis
| \
| 5.  $\neg(\exists x:T. ((A\ x) \Rightarrow B))$ 
| 6.  $B$ 
|  $\vdash \exists x:T. ((A\ x) \Rightarrow B)$ 
| |
BY RenameVar 'x' 2
| |
2.  $x: T$ 
|  $\vdash \exists x:T. ((A\ x) \Rightarrow B)$ 

```

|  
BY (InstConcl  $[\ulcorner x \urcorner]$ . THENA Auto)  
|  
 $\vdash (A\ x) \Rightarrow B$   
|  
BY (D 0 THENA Auto)  
|  
7. A x  
 $\vdash B$   
|  
BY Hypothesis