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├  $\forall [T:\text{Type}]. \forall [A,B:T \rightarrow \mathbb{P}]. \{\exists x:T. ((A\ x) \Rightarrow (B\ x)) \iff (\forall x:T. \{A\ x\}) \Rightarrow (\exists x:T. (B\ x))\}$ 
|
BY Auto
|
1. T: Type
2. A: T  $\rightarrow$   $\mathbb{P}$ 
3. B: T  $\rightarrow$   $\mathbb{P}$ 
├  $\{\exists x:T. ((A\ x) \Rightarrow (B\ x)) \iff (\forall x:T. \{A\ x\}) \Rightarrow (\exists x:T. (B\ x))\}$ 
|
BY RepeatFor 4 ((D 0 THENA Auto))
| \
| 4.  $\exists x:T. ((A\ x) \Rightarrow (B\ x))$ 
| ─  $\{(\forall x:T. \{A\ x\}) \Rightarrow (\exists x:T. (B\ x))\}$ 
| |
1 BY RepeatFor 2 ((D 0 THENA Auto))
| |
| 5.  $\forall x:T. \{A\ x\}$ 
| ─  $\{\exists x:T. (B\ x)\}$ 
| |
1 BY D 4
| |
| 4. x: T
| 5.  $(A\ x) \Rightarrow (B\ x)$ 
| 6.  $\forall x:T. \{A\ x\}$ 
| ─  $\{\exists x:T. (B\ x)\}$ 
| |
1 BY (InstHyp [x] 6. THENA Auto)
| |
| 7.  $\{A\ x\}$ 
| ─  $\{\exists x:T. (B\ x)\}$ 
| |
1 BY (ExposeClassical THENA Auto)
| |
| 7. A x
| ─  $\{\exists x:T. (B\ x)\}$ 
| |
1 BY (ElimClassical THENA Auto)
| |
| ─  $\exists x:T. (B\ x)$ 
| |
1 BY (InstConcl [x]. THENA Auto)
| |
| ─ B x
| |
1 BY D 5
| | \
| | 5.  $\forall x:T. \{A\ x\}$ 
| | 6. A x
| | ─ A x
| | |
1 2 BY Hypothesis
| | \
| | 5.  $\forall x:T. \{A\ x\}$ 
| | 6. A x
| | 7. B x
| | ─ B x

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| |
1 BY Hypothesis
|
| 4.  $(\forall x:T. \{A\ x\}) \Rightarrow (\exists x:T. (B\ x))$ 
|  $\vdash \{\exists x:T. ((A\ x) \Rightarrow (B\ x))\}$ 
|
| BY (ClassicalContradiction THENA Auto)
|
| 5.  $\neg(\exists x:T. ((A\ x) \Rightarrow (B\ x)))$ 
|  $\vdash \{\exists x:T. ((A\ x) \Rightarrow (B\ x))\}$ 
|
| BY D 4
| \
| | 4.  $\neg(\exists x:T. ((A\ x) \Rightarrow (B\ x)))$ 
| |  $\vdash \forall x:T. \{A\ x\}$ 
| |
| | 1 BY (D 0 THENA Auto)
| |
| | 5.  $x: T$ 
| |  $\vdash \{A\ x\}$ 
| |
| | 1 BY (ClassicalContradiction THENA Auto)
| |
| | 6.  $\neg(A\ x)$ 
| |  $\vdash \{A\ x\}$ 
| |
| | 1 BY D 4
| |
| | 4.  $x: T$ 
| | 5.  $\neg(A\ x)$ 
| |  $\vdash \exists x:T. ((A\ x) \Rightarrow (B\ x))$ 
| |
| | 1 BY (InstConcl [ $\lceil x \rceil$ ]. THENA Auto)
| |
| |  $\vdash (A\ x) \Rightarrow (B\ x)$ 
| |
| | 1 BY (D 0 THENA Auto)
| |
| | 6.  $A\ x$ 
| |  $\vdash B\ x$ 
| |
| | 1 BY D 5
| |
| | 5.  $A\ x$ 
| |  $\vdash A\ x$ 
| |
| 1 BY Hypothesis
|
| 4.  $\neg(\exists x:T. ((A\ x) \Rightarrow (B\ x)))$ 
| 5.  $\exists x:T. (B\ x)$ 
|  $\vdash \{\exists x:T. ((A\ x) \Rightarrow (B\ x))\}$ 
|
| BY D 5
|
| 5.  $x: T$ 
| 6.  $B\ x$ 
|  $\vdash \{\exists x:T. ((A\ x) \Rightarrow (B\ x))\}$ 

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|
BY (ElimClassical THENA Auto)
|
 $\vdash \exists x:T. ((A\ x) \Rightarrow (B\ x))$ 
|
BY (InstConcl [ $x$ ]. THENA Auto)
|
 $\vdash (A\ x) \Rightarrow (B\ x)$ 
|
BY (D 0 THENA Auto)
|
7.  $A\ x$ 
 $\vdash B\ x$ 
|
BY Hypothesis
```