```
⊢ ∀x:ℤ. (max_seg_sum(x;[x]) ∧ max_initseg_sum(x;[x]))
|
BY RepeatFor 2 ((D 0 THENA Auto))
|\
| 1. x: ℤ
| ⊢ max_seg_sum(x;[x])
| |
1 BY Unfold ‘max_seg_sum‘ 0
| |
| ⊢ (([x] = []) ∧ (x = 0))
| |  ∨ ((∃h:ℤ. ∃t:ℤ List. ([x] = [h / t]))
| |    ∧ (∀p:ℕ||[x]||. ∀q:{p..||[x]||⁻}.  (x ≥ seg_sum(p;q;[x]) ))
| |    ∧ (∃a:ℕ||[x]||. ∃b:{a..||[x]||⁻}. (x = seg_sum(a;b;[x]))))
| |
1 BY (OrRight THENA Auto)
| |
| ⊢ (∃h:ℤ. ∃t:ℤ List. ([x] = [h / t]))
| |  ∧ (∀p:ℕ||[x]||. ∀q:{p..||[x]||⁻}.  (x ≥ seg_sum(p;q;[x]) ))
| |  ∧ (∃a:ℕ||[x]||. ∃b:{a..||[x]||⁻}. (x = seg_sum(a;b;[x])))
| |
1 BY D 0
| |\
| | ⊢ ∃h:ℤ. ∃t:ℤ List. ([x] = [h / t])
| | |
1 2 BY (InstConcl [⌈x⌉;⌈[]⌉]· THEN Auto)
|  \
|   ⊢ (∀p:ℕ||[x]||. ∀q:{p..||[x]||⁻}.  (x ≥ seg_sum(p;q;[x]) ))
|   |  ∧ (∃a:ℕ||[x]||. ∃b:{a..||[x]||⁻}. (x = seg_sum(a;b;[x])))
|   |
1   BY D 0
|   |\
|   | ⊢ ∀p:ℕ||[x]||. ∀q:{p..||[x]||⁻}.  (x ≥ seg_sum(p;q;[x]) )
|   | |
1   2 BY RepeatFor 2 ((D 0 THENA Auto))·
|   | |
|   | 2. p: ℕ||[x]||
|   | 3. q: {p..||[x]||⁻}
|   | ⊢ x ≥ seg_sum(p;q;[x])
|   | |
1   2 BY (((Reduce 2 THEN Assert ⌈p = 0⌉· THEN Auto) THEN (HypSubst (-1) 0 THENA Auto))
|   | |   THEN (Reduce 3 THEN Assert ⌈q = 0⌉· THEN Auto)
|   | |   THEN (HypSubst (-1) 0 THENA Auto))
|   | |
|   | 2. p: ℕ1
|   | 3. q: {p..1⁻}
|   | 4. p = 0
|   | 5. q = 0
|   | ⊢ x ≥ seg_sum(0;0;[x])
|   | |
1   2 BY ((InstLemma ‘seg_sum0‘ [⌈0⌉;⌈[x]⌉]· THENA Auto) THEN (HypSubst (-1) 0 THENA Auto))
|   | |
|   | 6. seg_sum(0;0;[x]) = initseg_sum(0;[x])
|   | ⊢ x ≥ initseg_sum(0;[x])
```

```
|   | |
1   2 BY ((InstLemma ‘initseg_sum0‘ [⌈x⌉;⌈[]⌉]· THENA Auto) THEN (HypSubst (-1) 0 THENA Auto))
|   | |
|   | 7. initseg_sum(0;[x]) = x
|   | ⊢ x ≥ x
|   | |
1   2 BY Auto
|    \
|     ⊢ ∃a:ℕ||[x]||. ∃b:{a..||[x]||⁻}. (x = seg_sum(a;b;[x]))
|     |
1     BY (InstConcl [⌈0⌉;⌈0⌉]· THENA Auto)·
|     |
|     ⊢ x = seg_sum(0;0;[x])
|     |
1     BY ((InstLemma ‘seg_sum0‘ [⌈0⌉;⌈[x]⌉]· THENA Auto)
|     |   THEN (InstLemma ‘initseg_sum0‘ [⌈x⌉;⌈[]⌉]· THENA Auto)
|     |   )
|     |
|     2. seg_sum(0;0;[x]) = initseg_sum(0;[x])
|     3. initseg_sum(0;[x]) = x
|     ⊢ x = seg_sum(0;0;[x])
|     |
1     BY Auto
 \
  1. x: ℤ
  ⊢ max_initseg_sum(x;[x])
  |
  BY Unfold ‘max_initseg_sum‘ 0
  |
  ⊢ (([x] = []) ∧ (x = 0))
  |  ∨ ((∃h:ℤ. ∃t:ℤ List. ([x] = [h / t]))
  |    ∧ (∀r:ℕ||[x]||. (x ≥ initseg_sum(r;[x]) ))
  |    ∧ (∃c:ℕ||[x]||. (x = initseg_sum(c;[x]))))
  |
  BY (OrRight THENA Auto)
  |
  ⊢ (∃h:ℤ. ∃t:ℤ List. ([x] = [h / t]))
  |  ∧ (∀r:ℕ||[x]||. (x ≥ initseg_sum(r;[x]) ))
  |  ∧ (∃c:ℕ||[x]||. (x = initseg_sum(c;[x])))
  |
  BY D 0
  |\
  | ⊢ ∃h:ℤ. ∃t:ℤ List. ([x] = [h / t])
  | |
  1 BY (InstConcl [⌈x⌉;⌈[]⌉]· THEN Auto)
   \
    ⊢ (∀r:ℕ||[x]||. (x ≥ initseg_sum(r;[x]) )) ∧ (∃c:ℕ||[x]||. (x = initseg_sum(c;[x])))
    |
    BY D 0
    |\
    | ⊢ ∀r:ℕ||[x]||. (x ≥ initseg_sum(r;[x]) )
    | |
    1 BY (D 0 THENA Auto)
    | |
```

```
| 2. r: ℕ||[x]||
| ⊢ x ≥ initseg_sum(r;[x])
| |
1 BY ((Reduce 2 THEN Assert ⌈r = 0⌉⋅ THEN Auto) THEN (HypSubst (-1) 0 THENA Auto))
| |
| 2. r: ℕ1
| 3. r = 0
| ⊢ x ≥ initseg_sum(0;[x])
| |
1 BY (InstLemma ‘initseg_sum0‘ [⌈x⌉;⌈[]⌉]⋅ THENA Auto)
| |
| 4. initseg_sum(0;[x]) = x
| ⊢ x ≥ initseg_sum(0;[x])
| |
1 BY Auto
 \
  ⊢ ∃c:ℕ||[x]||. (x = initseg_sum(c;[x]))
  |
  BY (InstConcl [⌈0⌉]⋅ THENA Auto)
  |
  ⊢ x = initseg_sum(0;[x])
  |
  BY (InstLemma ‘initseg_sum0‘ [⌈x⌉;⌈[]⌉]⋅ THENA Auto)
  |
  2. initseg_sum(0;[x]) = x
  ⊢ x = initseg_sum(0;[x])
  |
  BY Auto
```