```
⊢ ∀L:ℤ List. (∃p:{ℤ × ℤ| let m,i = p in max_seg_sum(m;L) ∧ max_initseg_sum(i;L)})
|
BY (D 0 THENA Auto)
|
1. L: ℤ List
⊢ ∃p:{ℤ × ℤ| let m,i = p
|              in max_seg_sum(m;L) ∧ max_initseg_sum(i;L)}
|
BY ListInd 1
|\
| ⊢ ∃p:{ℤ × ℤ| let m,i = p
| |              in max_seg_sum(m;[]) ∧ max_initseg_sum(i;[])}
| |
1 BY ((InstConcl [⌈<0, 0>⌉]· THENA Auto) THEN Reduce 0)
| |
| ⊢ max_seg_sum(0;[]) ∧ max_initseg_sum(0;[])
| |
1 BY D 0
| |\
| | ⊢ max_seg_sum(0;[])
| | |
1 2 BY (Unfold ‘max_seg_sum‘ 0 THEN Auto)
|  \
|   ⊢ max_initseg_sum(0;[])
|   |
1   BY (Unfold ‘max_initseg_sum‘ 0 THEN Auto)
 \
  2. u: ℤ
  3. v: ℤ List
  4. ∃p:{ℤ × ℤ| let m,i = p
                in max_seg_sum(m;v) ∧ max_initseg_sum(i;v)}
  ⊢ ∃p:{ℤ × ℤ| let m,i = p
  |             in max_seg_sum(m;[u / v]) ∧ max_initseg_sum(i;[u / v])}
  |
  BY D 3
  |\
  | 3. ∃p:{ℤ × ℤ| let m,i = p
  |               in max_seg_sum(m;[]) ∧ max_initseg_sum(i;[])}
  | ⊢ ∃p:{ℤ × ℤ| let m,i = p
  | |             in max_seg_sum(m;[u]) ∧ max_initseg_sum(i;[u])}
  | |
  1 BY ((InstConcl [⌈<u, u>⌉]· THENA Auto) THEN Reduce 0)
  | |
  | ⊢ max_seg_sum(u;[u]) ∧ max_initseg_sum(u;[u])
  | |
  1 BY (InstLemma ‘maxsegsum_singleton‘ [⌈u⌉]· THEN Auto)
   \
    3. u1: ℤ
    4. v: ℤ List
    5. ∃p:{ℤ × ℤ| let m,i = p
                  in max_seg_sum(m;[u1 / v]) ∧ max_initseg_sum(i;[u1 / v])}
    ⊢ ∃p:{ℤ × ℤ| let m,i = p
    |             in max_seg_sum(m;[u; u1 / v]) ∧ max_initseg_sum(i;[u; u1 / v])}
```

```
|
BY (RepeatFor 2 (D 5) THEN Reduce (-1))
|
5. p1: ℤ
6. p2: ℤ
[7]. max_seg_sum(p1;[u1 / v]) ∧ max_initseg_sum(p2;[u1 / v])
⊢ ∃p:{ℤ × ℤ| let m,i = p
|             in max_seg_sum(m;[u; u1 / v]) ∧ max_initseg_sum(i;[u; u1 / v])}
|
BY ((InstConcl [⌈<imax(p1;imax(u;u + p2)), imax(u;u + p2)>⌉]· THENA Auto) THEN Reduce 0)
|
7. max_seg_sum(p1;[u1 / v]) ∧ max_initseg_sum(p2;[u1 / v])
⊢ max_seg_sum(imax(p1;imax(u;u + p2));[u; u1 / v])
|  ∧ max_initseg_sum(imax(u;u + p2);[u; u1 / v])
|
BY D 7
|
7. max_seg_sum(p1;[u1 / v])
8. max_initseg_sum(p2;[u1 / v])
⊢ max_seg_sum(imax(p1;imax(u;u + p2));[u; u1 / v])
|  ∧ max_initseg_sum(imax(u;u + p2);[u; u1 / v])
|
BY Assert ⌈max_initseg_sum(imax(u;u + p2);[u; u1 / v])
|          ∧ (max_initseg_sum(imax(u;u + p2);[u; u1 / v])
|            ⇒ max_seg_sum(imax(p1;imax(u;u + p2));[u; u1 / v]))⌉·
|\
| ⊢ max_initseg_sum(imax(u;u + p2);[u; u1 / v])
| |  ∧ (max_initseg_sum(imax(u;u + p2);[u; u1 / v])
| |    ⇒ max_seg_sum(imax(p1;imax(u;u + p2));[u; u1 / v]))
| |
1 BY D 0
| |\
| | ⊢ max_initseg_sum(imax(u;u + p2);[u; u1 / v])
| | |
1 2 BY Unfold ‘max_initseg_sum‘ 0
| | |
| | ⊢ (([u; u1 / v] = []) ∧ (imax(u;u + p2) = 0))
| | |  ∨ ((∃h:ℤ. ∃t:ℤ List. ([u; u1 / v] = [h / t]))
| | |    ∧ (∀r:ℕ||[u; u1 / v]||. (imax(u;u + p2) ≥ initseg_sum(r;[u; u1 / v]) ))
| | |    ∧ (∃c:ℕ||[u; u1 / v]||. (imax(u;u + p2) = initseg_sum(c;[u; u1 / v]))))
| | |
1 2 BY (OrRight THENA Auto)
| | |
| | ⊢ (∃h:ℤ. ∃t:ℤ List. ([u; u1 / v] = [h / t]))
| | |  ∧ (∀r:ℕ||[u; u1 / v]||. (imax(u;u + p2) ≥ initseg_sum(r;[u; u1 / v]) ))
| | |  ∧ (∃c:ℕ||[u; u1 / v]||. (imax(u;u + p2) = initseg_sum(c;[u; u1 / v])))
| | |
1 2 BY D 0
| | |\
| | | ⊢ ∃h:ℤ. ∃t:ℤ List. ([u; u1 / v] = [h / t])
| | | |
1 2 3 BY (InstConcl [⌈u⌉;⌈[u1 / v]⌉]· THEN Auto)
| |  \
| |   ⊢ (∀r:ℕ||[u; u1 / v]||. (imax(u;u + p2) ≥ initseg_sum(r;[u; u1 / v]) ))
```

```
| |   |  ∧ (∃c:ℕ||[u; u1 / v]||. (imax(u;u + p2) = initseg_sum(c;[u; u1 / v])))
| |   |
1 2   BY D 0
| |   |\
| |   | ⊢ ∀r:ℕ||[u; u1 / v]||. (imax(u;u + p2) ≥ initseg_sum(r;[u; u1 / v]) )
| |   | |
1 2   3 BY (D 0 THENA Auto)
| |   | |
| |   | 9. r: ℕ||[u; u1 / v]||
| |   | ⊢ imax(u;u + p2) ≥ initseg_sum(r;[u; u1 / v])
| |   | |
1 2   3 BY (Decide ⌈r = 0⌉· THENA Auto)
| |   | |\
| |   | | 10. r = 0
| |   | | ⊢ imax(u;u + p2) ≥ initseg_sum(r;[u; u1 / v])
| |   | | |
1 2   3 4 BY (HypSubst 10 0 THENA Auto)
| |   | | |
| |   | | ⊢ imax(u;u + p2) ≥ initseg_sum(0;[u; u1 / v])
| |   | | |
1 2   3 4 BY ((InstLemma `initseg_sum0` [⌈u⌉;⌈[u1 / v]⌉]· THENA Auto)
| |   | | |   THEN (HypSubst (-1) 0 THENA Auto)
| |   | | |   )
| |   | | |
| |   | | 11. initseg_sum(0;[u; u1 / v]) = u
| |   | | ⊢ imax(u;u + p2) ≥ u
| |   | | |
1 2   3 4 BY (InstLemma `imax_ge_left` [⌈u⌉;⌈u + p2⌉]· THEN Auto)
| |   |  \
| |   |   10. ¬(r = 0)
| |   |   ⊢ imax(u;u + p2) ≥ initseg_sum(r;[u; u1 / v])
| |   |   |
1 2   3   BY ((InstLemma `initseg_sum_shift` [⌈r⌉;⌈u⌉;⌈[u1 / v]⌉]· THENA Auto)
| |   |   |   THEN (HypSubst (-1) 0 THENA Auto)
| |   |   |   )
| |   |   |
| |   |   11. initseg_sum(r;[u; u1 / v]) = (u + initseg_sum(r - 1;[u1 / v]))
| |   |   ⊢ imax(u;u + p2) ≥ (u + initseg_sum(r - 1;[u1 / v]))
| |   |   |
1 2   3   BY (D 8 THEN Auto)
| |   |   |
| |   |   8. ∃h:ℤ. ∃t:ℤ List. ([u1 / v] = [h / t])
| |   |   9. ∀r:ℕ||[u1 / v]||. (p2 ≥ initseg_sum(r;[u1 / v]) )
| |   |   10. ∃c:ℕ||[u1 / v]||. (p2 = initseg_sum(c;[u1 / v]))
| |   |   11. r: ℕ||[u; u1 / v]||
| |   |   12. ¬(r = 0)
| |   |   13. initseg_sum(r;[u; u1 / v]) = (u + initseg_sum(r - 1;[u1 / v]))
| |   |   ⊢ imax(u;u + p2) ≥ (u + initseg_sum(r - 1;[u1 / v]))
| |   |   |
1 2   3   BY (InstHyp [⌈r - 1⌉] 9· THENA Auto)
| |   |   |
| |   |   14. p2 ≥ initseg_sum(r - 1;[u1 / v])
| |   |   ⊢ imax(u;u + p2) ≥ (u + initseg_sum(r - 1;[u1 / v]))
| |   |   |
```

```
1 2   3   BY (InstLemma ‘imax_ge_right‘ [⌈u⌉;⌈u + p2⌉]· THENA Auto)
| |   |   |
| |   |   15. imax(u;u + p2) ≥ (u + p2)
| |   |   ⊢ imax(u;u + p2) ≥ (u + initseg_sum(r - 1;[u1 / v]))
| |   |   |
1 2   3   BY Auto’
| |    \
| |     ⊢ ∃c:ℕ||[u; u1 / v]||. (imax(u;u + p2) = initseg_sum(c;[u; u1 / v]))
| |     |
1 2     BY (Decide ⌈u ≤ (u + p2)⌉· THENA Auto)
| |     |\
| |     | 9. u ≤ (u + p2)
| |     | ⊢ ∃c:ℕ||[u; u1 / v]||. (imax(u;u + p2) = initseg_sum(c;[u; u1 / v]))
| |     | |
1 2     3 BY (D 8 THEN Auto)
| |     | |
| |     | 8. ∃h:ℤ. ∃t:ℤ List. ([u1 / v] = [h / t])
| |     | 9. ∀r:ℕ||[u1 / v]||. (p2 ≥ initseg_sum(r;[u1 / v]) )
| |     | 10. ∃c:ℕ||[u1 / v]||. (p2 = initseg_sum(c;[u1 / v]))
| |     | 11. u ≤ (u + p2)
| |     | ⊢ ∃c:ℕ||[u; u1 / v]||. (imax(u;u + p2) = initseg_sum(c;[u; u1 / v]))
| |     | |
1 2     3 BY (D 10 THEN (InstConcl [⌈c + 1⌉]· THENA Auto))
| |     | |
| |     | 10. c: ℕ||[u1 / v]||
| |     | 11. p2 = initseg_sum(c;[u1 / v])
| |     | 12. u ≤ (u + p2)
| |     | ⊢ imax(u;u + p2) = initseg_sum(c + 1;[u; u1 / v])
| |     | |
1 2     3 BY ((InstLemma ‘initseg_sum_shift‘ [⌈c + 1⌉;⌈u⌉;⌈[u1 / v]⌉]· THENA Auto)
| |     | |   THEN (HypSubst (-1) 0 THENA Auto)
| |     | |   )
| |     | |
| |     | 13. initseg_sum(c + 1;[u; u1 / v]) = (u + initseg_sum((c + 1) - 1;[u1 / v]))
| |     | ⊢ imax(u;u + p2) = (u + initseg_sum((c + 1) - 1;[u1 / v]))
| |     | |
1 2     3 BY (((Assert ⌈((c + 1) - 1) = c⌉· THEN Auto) THEN (HypSubst (-1) 0 THENA Auto))
| |     | |   THEN (RevHypSubst 11 0 THENA Auto)
| |     | |   )
| |     | |
| |     | 14. ((c + 1) - 1) = c
| |     | ⊢ imax(u;u + p2) = (u + p2)
| |     | |
1 2     3 BY ((Unfold ‘imax‘ 0 THEN RepeatFor 2 ((CallByValueReduce 0 THENA Auto))) THEN Auto)
| |      \
| |       9. ¬(u ≤ (u + p2))
| |       ⊢ ∃c:ℕ||[u; u1 / v]||. (imax(u;u + p2) = initseg_sum(c;[u; u1 / v]))
| |       |
1 2       BY (InstConcl [⌈0⌉]· THENA Auto)
| |       |
| |       ⊢ imax(u;u + p2) = initseg_sum(0;[u; u1 / v])
| |       |
1 2       BY ((InstLemma ‘initseg_sum0‘ [⌈u⌉;⌈[u1 / v]⌉]· THENA Auto)
| |       |   THEN (HypSubst (-1) 0 THENA Auto)
```

```
| |       |   )
| |       |
| |       10. initseg_sum(0;[u; u1 / v]) = u
| |       ⊢ imax(u;u + p2) = u
| |       |
1 2       BY ((Unfold ‘imax‘ 0 THEN RepeatFor 2 ((CallByValueReduce 0 THENA Auto))) THEN Auto)
|  \
|   ⊢ max_initseg_sum(imax(u;u + p2);[u; u1 / v])
|   |  ⇒ max_seg_sum(imax(p1;imax(u;u + p2));[u; u1 / v])
|   |
1   BY (D 0 THENA Auto)
|   |
|   9. max_initseg_sum(imax(u;u + p2);[u; u1 / v])
|   ⊢ max_seg_sum(imax(p1;imax(u;u + p2));[u; u1 / v])
|   |
1   BY Unfold ‘max_seg_sum‘ 0
|   |
|   ⊢ (([u; u1 / v] = []) ∧ (imax(p1;imax(u;u + p2)) = 0))
|   |  ∨ ((∃h:ℤ. ∃t:ℤ List. ([u; u1 / v] = [h / t]))
|   |    ∧ (∀p:ℕ||[u; u1 / v]||. ∀q:{p..||[u; u1 / v]||⁻}.
|   |         (imax(p1;imax(u;u + p2)) ≥ seg_sum(p;q;[u; u1 / v]) ))
|   |    ∧ (∃a:ℕ||[u; u1 / v]||
|   |        ∃b:{a..||[u; u1 / v]||⁻}. (imax(p1;imax(u;u + p2)) = seg_sum(a;b;[u; u1 / v]))))
|   |
1   BY (OrRight THENA Auto)
|   |
|   ⊢ (∃h:ℤ. ∃t:ℤ List. ([u; u1 / v] = [h / t]))
|   |  ∧ (∀p:ℕ||[u; u1 / v]||. ∀q:{p..||[u; u1 / v]||⁻}.
|   |       (imax(p1;imax(u;u + p2)) ≥ seg_sum(p;q;[u; u1 / v]) ))
|   |  ∧ (∃a:ℕ||[u; u1 / v]||
|   |      ∃b:{a..||[u; u1 / v]||⁻}. (imax(p1;imax(u;u + p2)) = seg_sum(a;b;[u; u1 / v])))
|   |
1   BY D 0
|   |\
|   | ⊢ ∃h:ℤ. ∃t:ℤ List. ([u; u1 / v] = [h / t])
|   | |
1   2 BY (InstConcl [⌈u⌉;⌈[u1 / v]⌉]· THEN Auto)
|    \
|     ⊢ (∀p:ℕ||[u; u1 / v]||. ∀q:{p..||[u; u1 / v]||⁻}.
|     |     (imax(p1;imax(u;u + p2)) ≥ seg_sum(p;q;[u; u1 / v]) ))
|     |  ∧ (∃a:ℕ||[u; u1 / v]||
|     |      ∃b:{a..||[u; u1 / v]||⁻}. (imax(p1;imax(u;u + p2)) = seg_sum(a;b;[u; u1 / v])))
|     |
1     BY D 0
|     |\
|     | ⊢ ∀p:ℕ||[u; u1 / v]||. ∀q:{p..||[u; u1 / v]||⁻}.
|     | |    (imax(p1;imax(u;u + p2)) ≥ seg_sum(p;q;[u; u1 / v]) )
|     | |
1     2 BY RepeatFor 2 ((D 0 THENA Auto))
|     | |
|     | 10. p: ℕ||[u; u1 / v]||
|     | 11. q: {p..||[u; u1 / v]||⁻}
|     | ⊢ imax(p1;imax(u;u + p2)) ≥ seg_sum(p;q;[u; u1 / v])
|     | |
```

```
1     2 BY (Decide ⌈p = 0⌉· THENA Auto)·
|     | |\
|     | | 12. p = 0
|     | | ⊢ imax(p1;imax(u;u + p2)) ≥ seg_sum(p;q;[u; u1 / v])
|     | | |
1     2 3 BY (HypSubst 12 0 THENA Auto)
|     | | |
|     | | ⊢ imax(p1;imax(u;u + p2)) ≥ seg_sum(0;q;[u; u1 / v])
|     | | |
1     2 3 BY ((InstLemma ‘seg_sum0‘ [⌈q⌉;⌈[u; u1 / v]⌉]· THENA Auto)
|     | | |   THEN (HypSubst (-1) 0 THENA Auto)
|     | | |   )
|     | | |
|     | | 13. seg_sum(0;q;[u; u1 / v]) = initseg_sum(q;[u; u1 / v])
|     | | ⊢ imax(p1;imax(u;u + p2)) ≥ initseg_sum(q;[u; u1 / v])
|     | | |
1     2 3 BY (D 9 THEN Auto)
|     | | |
|     | | 9. ∃h:ℤ. ∃t:ℤ List. ([u; u1 / v] = [h / t])
|     | | 10. ∀r:ℕ||[u; u1 / v]||. (imax(u;u + p2) ≥ initseg_sum(r;[u; u1 / v]) )
|     | | 11. ∃c:ℕ||[u; u1 / v]||. (imax(u;u + p2) = initseg_sum(c;[u; u1 / v]))
|     | | 12. p: ℕ||[u; u1 / v]||
|     | | 13. q: {p..||[u; u1 / v]||⁻}
|     | | 14. p = 0
|     | | 15. seg_sum(0;q;[u; u1 / v]) = initseg_sum(q;[u; u1 / v])
|     | | ⊢ imax(p1;imax(u;u + p2)) ≥ initseg_sum(q;[u; u1 / v])
|     | | |
1     2 3 BY (InstHyp [⌈q⌉] 10· THENA Auto)
|     | | |
|     | | 16. imax(u;u + p2) ≥ initseg_sum(q;[u; u1 / v])
|     | | ⊢ imax(p1;imax(u;u + p2)) ≥ initseg_sum(q;[u; u1 / v])
|     | | |
1     2 3 BY (InstLemma ‘imax_ge_right‘ [⌈p1⌉;⌈imax(u;u + p2)⌉]· THENA Auto)
|     | | |
|     | | 17. imax(p1;imax(u;u + p2)) ≥ imax(u;u + p2)
|     | | ⊢ imax(p1;imax(u;u + p2)) ≥ initseg_sum(q;[u; u1 / v])
|     | | |
1     2 3 BY Auto
|     |  \
|     |   12. ¬(p = 0)
|     |   ⊢ imax(p1;imax(u;u + p2)) ≥ seg_sum(p;q;[u; u1 / v])
|     |   |
1     2   BY ((InstLemma ‘seg_sum_shift‘ [⌈p⌉;⌈q⌉;⌈[u; u1 / v]⌉]· THENA Auto)
|     |   |   THEN (HypSubst (-1) 0 THENA Auto)
|     |   |   THEN Reduce 0)
|     |   |
|     |   13. seg_sum(p;q;[u; u1 / v]) = seg_sum(p - 1;q - 1;tl([u; u1 / v]))
|     |   ⊢ imax(p1;imax(u;u + p2)) ≥ seg_sum(p - 1;q - 1;[u1 / v])
|     |   |
1     2   BY (D 7 THEN Auto)
|     |   |
|     |   7. ∃h:ℤ. ∃t:ℤ List. ([u1 / v] = [h / t])
|     |   8. ∀p:ℕ||[u1 / v]||. ∀q:{p..||[u1 / v]||⁻}.  (p1 ≥ seg_sum(p;q;[u1 / v]) )
|     |   9. ∃a:ℕ||[u1 / v]||. ∃b:{a..||[u1 / v]||⁻}. (p1 = seg_sum(a;b;[u1 / v]))
```

```
|      |    10. max_initseg_sum(p2;[u1 / v])
|      |    11. max_initseg_sum(imax(u;u + p2);[u; u1 / v])
|      |    12. p: ℕ||[u; u1 / v]||
|      |    13. q: {p..||[u; u1 / v]||⁻}
|      |    14. ¬(p = 0)
|      |    15. seg_sum(p;q;[u; u1 / v]) = seg_sum(p - 1;q - 1;tl([u; u1 / v]))
|      |    ⊢ imax(p1;imax(u;u + p2)) ≥ seg_sum(p - 1;q - 1;[u1 / v])
|      |    |
1      2    BY (InstHyp [⌈p - 1⌉;⌈q - 1⌉] 8· THENA Auto)
|      |    |
|      |    16. p1 ≥ seg_sum(p - 1;q - 1;[u1 / v])
|      |    ⊢ imax(p1;imax(u;u + p2)) ≥ seg_sum(p - 1;q - 1;[u1 / v])
|      |    |
1      2    BY (InstLemma `imax_ge_left` [⌈p1⌉;⌈imax(u;u + p2)⌉]· THENA Auto)
|      |    |
|      |    17. imax(p1;imax(u;u + p2)) ≥ p1
|      |    ⊢ imax(p1;imax(u;u + p2)) ≥ seg_sum(p - 1;q - 1;[u1 / v])
|      |    |
1      2    BY Auto
|       \
|        ⊢ ∃a:ℕ||[u; u1 / v]||
|        |   ∃b:{a..||[u; u1 / v]||⁻}. (imax(p1;imax(u;u + p2)) = seg_sum(a;b;[u; u1 / v]))
|        |
1        BY (Decide ⌈p1 ≤ imax(u;u + p2)⌉· THENA Auto)·
|        |\
|        | 10. p1 ≤ imax(u;u + p2)
|        | ⊢ ∃a:ℕ||[u; u1 / v]||
|        | |   ∃b:{a..||[u; u1 / v]||⁻}. (imax(p1;imax(u;u + p2)) = seg_sum(a;b;[u; u1 / v]))
|        | |
1        2 BY (D 9 THEN Auto)
|        | |
|        | 9. ∃h:ℤ. ∃t:ℤ List. ([u; u1 / v] = [h / t])
|        | 10. ∀r:ℕ||[u; u1 / v]||. (imax(u;u + p2) ≥ initseg_sum(r;[u; u1 / v]) )
|        | 11. ∃c:ℕ||[u; u1 / v]||. (imax(u;u + p2) = initseg_sum(c;[u; u1 / v]))
|        | 12. p1 ≤ imax(u;u + p2)
|        | ⊢ ∃a:ℕ||[u; u1 / v]||
|        | |   ∃b:{a..||[u; u1 / v]||⁻}. (imax(p1;imax(u;u + p2)) = seg_sum(a;b;[u; u1 / v]))
|        | |
1        2 BY (D 11 THEN (InstConcl [⌈0⌉;⌈c⌉]· THENA Auto))
|        | |
|        | 11. c: ℕ||[u; u1 / v]||
|        | 12. imax(u;u + p2) = initseg_sum(c;[u; u1 / v])
|        | 13. p1 ≤ imax(u;u + p2)
|        | ⊢ imax(p1;imax(u;u + p2)) = seg_sum(0;c;[u; u1 / v])
|        | |
1        2 BY ((InstLemma `seg_sum0` [⌈c⌉;⌈[u; u1 / v]⌉]· THENA Auto)
|        | |   THEN (HypSubst (-1) 0 THENA Auto)
|        | |   )
|        | |
|        | 14. seg_sum(0;c;[u; u1 / v]) = initseg_sum(c;[u; u1 / v])
|        | ⊢ imax(p1;imax(u;u + p2)) = initseg_sum(c;[u; u1 / v])
|        | |
1        2 BY (RevHypSubst 12 0 THENA Auto)
|        | |
```

```
|       | ⊢ imax(p1;imax(u;u + p2)) = imax(u;u + p2)
|       | |
1       2 BY (Unfold ‘imax‘ 0 THEN RepeatFor 4 ((CallByValueReduce 0 THENA Auto)) THEN Auto)
|        \
|         10. ¬(p1 ≤ imax(u;u + p2))
|         ⊢ ∃a:ℕ||[u; u1 / v]||
|         |   ∃b:{a..||[u; u1 / v]||⁻}. (imax(p1;imax(u;u + p2)) = seg_sum(a;b;[u; u1 / v]))
|         |
1         BY (D 7 THEN Auto)
|         |
|         7. ∃h:ℤ. ∃t:ℤ List. ([u1 / v] = [h / t])
|         8. ∀p:ℕ||[u1 / v]||. ∀q:{p..||[u1 / v]||⁻}.  (p1 ≥ seg_sum(p;q;[u1 / v]) )
|         9. ∃a:ℕ||[u1 / v]||. ∃b:{a..||[u1 / v]||⁻}. (p1 = seg_sum(a;b;[u1 / v]))
|         10. max_initseg_sum(p2;[u1 / v])
|         11. max_initseg_sum(imax(u;u + p2);[u; u1 / v])
|         12. ¬(p1 ≤ imax(u;u + p2))
|         ⊢ ∃a:ℕ||[u; u1 / v]||
|         |   ∃b:{a..||[u; u1 / v]||⁻}. (imax(p1;imax(u;u + p2)) = seg_sum(a;b;[u; u1 / v]))
|         |
1         BY (D 9 THEN D 10)
|         |
|         9. a: ℕ||[u1 / v]||
|         10. b: {a..||[u1 / v]||⁻}
|         11. p1 = seg_sum(a;b;[u1 / v])
|         12. max_initseg_sum(p2;[u1 / v])
|         13. max_initseg_sum(imax(u;u + p2);[u; u1 / v])
|         14. ¬(p1 ≤ imax(u;u + p2))
|         ⊢ ∃a:ℕ||[u; u1 / v]||
|         |   ∃b:{a..||[u; u1 / v]||⁻}. (imax(p1;imax(u;u + p2)) = seg_sum(a;b;[u; u1 / v]))
|         |
1         BY (InstConcl [⌈a + 1⌉;⌈b + 1⌉]· THENA Auto)·
|         |
|         ⊢ imax(p1;imax(u;u + p2)) = seg_sum(a + 1;b + 1;[u; u1 / v])
|         |
1         BY ((InstLemma ‘seg_sum_shift‘ [⌈a + 1⌉;⌈b + 1⌉;⌈[u; u1 / v]⌉]· THENA Auto)
|         |   THEN (HypSubst (-1) 0 THENA Auto)
|         |   THEN Reduce 0)
|         |
|         15. seg_sum(a + 1;b + 1;[u; u1 / v])
|             = seg_sum((a + 1) - 1;(b + 1) - 1;tl([u; u1 / v]))
|         ⊢ imax(p1;imax(u;u + p2)) = seg_sum((a + 1) - 1;(b + 1) - 1;[u1 / v])
|         |
1         BY ((Assert ⌈(((a + 1) - 1) = a) ∧ (((b + 1) - 1) = b)⌉· THEN Auto)
|         |   THEN (HypSubst (-1) 0 THENA Auto)
|         |   THEN (HypSubst (-2) 0 THENA Auto))
|         |
|         16. ((a + 1) - 1) = a
|         17. ((b + 1) - 1) = b
|         ⊢ imax(p1;imax(u;u + p2)) = seg_sum(a;b;[u1 / v])
|         |
1         BY (RevHypSubst 11 0 THENA Auto)
|         |
|         ⊢ imax(p1;imax(u;u + p2)) = p1
|         |
```

```
1         BY ((Unfold `imax` 0 THEN RepeatFor 4 ((CallByValueReduce 0 THENA Auto))) THEN Auto)
 \
  9. max_initseg_sum(imax(u;u + p2);[u; u1 / v])
     ∧ (max_initseg_sum(imax(u;u + p2);[u; u1 / v])
       ⇒ max_seg_sum(imax(p1;imax(u;u + p2));[u; u1 / v]))
  ⊢ max_seg_sum(imax(p1;imax(u;u + p2));[u; u1 / v])
  |  ∧ max_initseg_sum(imax(u;u + p2);[u; u1 / v])
  |
  BY Auto
```

Extract:

```
λL.letrec maxsegsum(L) =
  if L = [] then <0, 0> otherwise let u,v = L in
    if v = [] then <u, u> otherwise let u1,v1 = v in
      let p1,p2 = maxsegsum <u1, v1> in
      <imax(p1;imax(u;u + p2)), imax(u;u + p2)> in
  maxsegsum(L)
```