```
⊢ ∀n,m:ℕ.  (∃g:{ℕ| GCD(m;n;g)})
|
BY (D 0 THENA Auto)
|
1. n: ℕ
⊢ ∀m:ℕ. (∃g:{ℕ| GCD(m;n;g)})
|
BY (GeneralNatInd 1 THENA Auto)
|
2. ∀n1:ℕn. ∀m:ℕ.  (∃g:{ℕ| GCD(m;n1;g)})
⊢ ∀m:ℕ. (∃g:{ℕ| GCD(m;n;g)})
|
BY (D 0 THENA Auto)
|
3. m: ℕ
⊢ ∃g:{ℕ| GCD(m;n;g)}
|
BY (Decide ⌈n = 0⌉· THENA Auto)
|\
| 4. n = 0
| ⊢ ∃g:{ℕ| GCD(m;n;g)}
| |
1 BY (SqHypSubst 4 0 THENA Auto)
| |
| ⊢ ∃g:{ℕ| GCD(m;0;g)}
| |
1 BY (InstConcl [⌈m⌉]· THENA Auto)
| |
| ⊢ GCD(m;0;m)
| |
1 BY (InstLemma ‘gcd_p_zero‘ [⌈m⌉]· THENA Auto)
| |
| 5. GCD(m;0;m)
| ⊢ GCD(m;0;m)
| |
1 BY NthHyp 5
 \
  4. ¬(n = 0)
  ⊢ ∃g:{ℕ| GCD(m;n;g)}
  |
  BY (InstHyp [⌈m rem n⌉;⌈n⌉] 2· THENA Auto)
  |\
  | ⊢ (m rem n) < n
  | |
  1 BY (InstLemma ‘rem_bounds_1‘ [⌈m⌉;⌈n⌉]· THENA Auto)
  | |
  | 5. (0 ≤ (m rem n)) ∧ ((m rem n) < n)
  | ⊢ (m rem n) < n
  | |
  1 BY Auto
   \
    5. ∃g:{ℕ| GCD(n;m rem n;g)}
    ⊢ ∃g:{ℕ| GCD(m;n;g)}
```

```
|
BY D 5
|
5. g: ℕ
[6]. GCD(n;m rem n;g)
⊢ ∃g:{ℕ| GCD(m;n;g)}
|
BY (InstConcl [⌈g⌉]· THENA Auto)
|
6. GCD(n;m rem n;g)
⊢ GCD(m;n;g)
|
BY (InstLemma ‘div_rem_gcd_anne‘ [⌈m⌉;⌈n⌉;⌈g⌉]· THENA Auto)
|
7. GCD(m;n;g) ⟺ GCD(n;m rem n;g)
⊢ GCD(m;n;g)
|
BY Auto
```

Extract:

```
λn. letrec gcd(n) =
  λm. if n = 0  then m
      else (gcd (m rem n) n)
  in gcd(n)
```